Using the limit definition of the definite integral, and right endpoints, evaluate $\int_{-2}^{1}(4x+1)\,dx$. **SCORE: ____ / 15 PTS**

NOTE: Solutions using any other method will earn 0 points.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left(4\left(-2+\frac{3i}{n}\right)+1\right)\frac{3}{n}$$ (1) (1) (2) (2)

$$=\lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}\left(-7+\frac{12i}{n}\right)$$ (2)

$$=\lim_{n\to\infty}\frac{3}{n}\left(\sum_{i=1}^{n}-7+\frac{12}{n}\sum_{i=1}^{n}i\right)$$ (1)

$$=\lim_{n\to\infty}\frac{3}{n}\left(-7n+\frac{12}{n}\cdot\frac{n(n+1)}{2}\right)$$ (1) (1)

$$=\lim_{n\to\infty}\left(-21+\frac{18(n+1)}{n}\right)$$ (1)

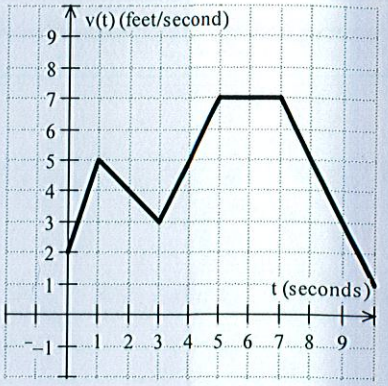
$$=-21+18$$ (1)

$$=-3$$ (1)

(1) FOR WRITING $\lim_{n\to\infty}$ ON EVERY LINE

A person's velocity as a function of time is shown in the graph on the right. **SCORE: _____ / 5 PTS**

NOTE: The graph is a piecewise defined function whose "pieces" are all linear functions.



[a] Estimate the distance the person travelled from time $t = 2$ second to $t = 10$ seconds using four subintervals and left endpoints.

$\Delta t = \frac{10-2}{4} = 2$

$v(2)\Delta t + v(4)\Delta t + v(6)\Delta t + v(8)\Delta t$

$= 4 \cdot 2 + 5 \cdot 2 + 7 \cdot 2 + 5 \cdot 2 = 42$ ft

[b] Find the exact distance the person travelled from time $t = 2$ second to $t = 10$ seconds.

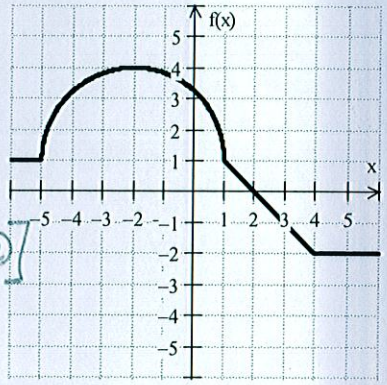
$\frac{1}{2}(4+3)(1) + \frac{1}{2}(3+7)(2) + 7(2) + \frac{1}{2}(7+1)(3)$

$= \frac{7}{2} + 10 + 14 + 12$ (½) POINT EACH

$= 39\frac{1}{2}$ ft

The graph of function f is shown on the right. **SCORE: _____ / 5 PTS**

The graph consists of a horizontal line, a semi-circle with center (-2, 1), a diagonal line and a horizontal line.



[a] Evaluate $\int_5^1 f(x)\,dx$.

(1) POINT EACH

$= -\int_1^5 f(x)\,dx$

$= -\left[\int_1^2 f(x)\,dx + \int_2^5 f(x)\,dx\right] = -\left[\frac{1}{2}(1)(1) + \frac{1}{2}(3+1)(2)\right]$

[b] Evaluate $\int_{-4}^{5} f(x)\,dx$.

$= -\left[\frac{1}{2} - 4\right] = \frac{7}{2}$

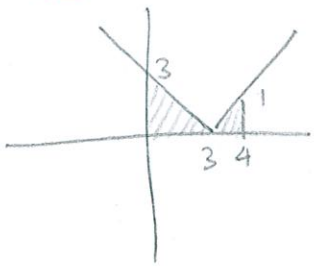
Evaluate $\int_0^4 (|x-3| + \sqrt{16-x^2})\,dx$ using the properties of definite integrals **SCORE: _____ / 5 PTS**

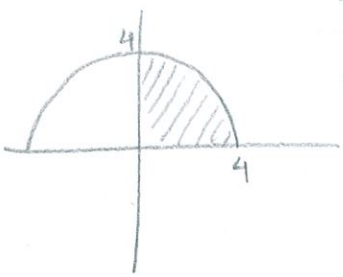
and interpreting in terms of area.

$= \int_0^4 |x-3|\,dx + \int_0^4 \sqrt{16-x^2}\,dx = \frac{1}{2}(3)(3) + \frac{1}{2}(1)(1) + \frac{1}{4}\pi(4)^2$

$= \frac{9}{2} + \frac{1}{2} + 4\pi$

$= 5 + 4\pi$





(1) POINT EACH